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Three Dimensional Analytical Model for the Dispersion of Air Pollutants Emitted from Elevated Point Source with Mesoscale Wind and Removal Mechanism

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This paper reports the method of solving three dimensional atmospheric diffusion equation by Fourier transform technique, variable separable method and series solution. In this model we consider the consequences of local wind known as mesoscale wind with large scale wind and removal mechanism on distribution of air impurities released from raised point source in a civic zone. The outcome of this model have been interpreted for the downwind, crosswind and vertical distances. We observed that the congregation profile decreases with increase in downwind, crosswind and vertical distance. Also the concentration of pollutants decreases with increase of removal rate.

Keywords: Analytical Method, Mesoscale Wind, Point Source, Removal Mechanism, Pollutant Dispersion.

RESEARCH ARTICLE

1. INTRODUCTION

Now a days majority of the natural resources are contaminated. The rapid increase in population, transportation and industrialization are the main reasons for the increase of all types of hazardous waste in the environs. The control of air pollution is one of the difficult tasks facing an environmental manager. Both soil and water can be confined and gathered at one place while air cannot be gathered and confined in one place. Hence the control technologies for air pollution have to be at the source before the release of pollutants in to the air. It is very necessary to understand the physical phenomenon associated with the aerosol impurities distribution to preserve environment from pollutants. We need to observe air quality constantly to reduce the air impurities liberation to the environment. Therefore, precise modeling of air impurities distribution near the ear this notable.

Urban heat island produces local or mesoscale wind that have to be taken into an account along with the large scale wind to predict the concentration. Griffiths[1] noted that huge scale air current is not sufficient to forecasts air pollution in civic zone. Verma[2] have given model on distribution of air pollutants by considering the constant air current velocity, Varma et al.[3] have studied distribution of air contaminants with varied wind velocity, Sharan et al.[4,5] have outline the mathematical modeling framework of meteorological distribution. These are analytical models but they have not considered the effect of mesoscale wind in civic zone. Chandler[6] pointed out that near the centre of warm islet the erect mixing would be increased by local wind. Dilley and Yen[7] have presented air pollution model for the spreading of air impurities in the presence of mesoscale wind for a line source. Venkatachalappa et al.[8] developed the time dependent numerical model of air impurities due to area source by considering the variable wind velocity, eddy diffusivity and chemical reaction. Pandurangappa and Lakshminarayanachari[9] have given numerical model to observe the consequences of mesoscale air current on the dispensation of air pollutants. Lakshminarayanachari et al.[10,11] developed the numerical model to study the dispensation of pollutants emitted from civic area source with removal mechanism and it the the two dimensional model. However these works did not deal with the consequences of mesoscale air current with removal mechanism for the point source. In view of this, we develop an analytical model to study the dispensation of air impurities liberated from raised point source in the presence of mesoscale wind and removal mechanism for the civic warm islet.

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2. MODEL FORMULATION

The governing equation is

$$U(x)\frac{\partial C}{\partial x}+w(z)\frac{\partial C}{\partial z}=k_y\frac{\partial^2 C}{\partial y^2}+k_z\frac{\partial^2 C}{\partial z^2}-\alpha C \qquad (1)$$

Where $C=C(x, y, z)$ is the mean congregation of impurities in the aerosphere k_y, k_z are the eddy diffusivity terms in y, z directions respectively. Typically $k_y > k_z$ in the aerosphere. α is the removal rate of the air impurities due to some legitimate system.

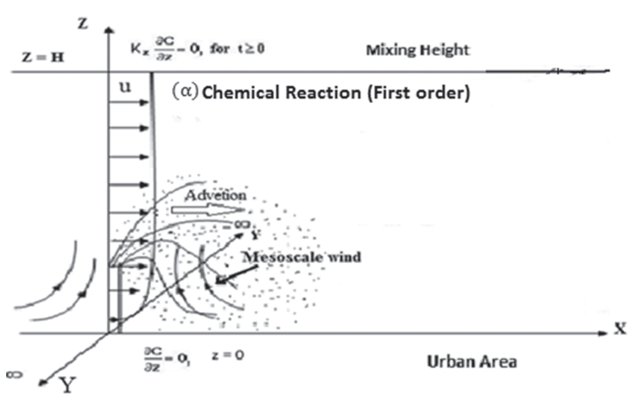
$U(x)$ is the wind velocity in the x direction which varies with the downwind distance. It is assumed in the form

$$U(x)=U_0(1-ax)$$

Where U_0 is the mean wind velocity.

$W(z)=U_0az$ is the mean wind speed in the z direction.

The corporal problem consists of a point source raised at a distance h_s mts from the ground. We suppose that the air impurities are emitted at steady speed from the point reference. The pollutants are transited parallel to huge scale air current and horizontally plus vertically by the local air current called mesoscale wind. The centre of warm islet is considered at origin, $(x=0, y=0)$. The congregation distribution was computed up to the preferred downwind length $l=5$ km i.e., $0\le x\le l$.



The boundary conditions for the Eq. (1) are taken as follows:

(1) The pollutant is liberated from lofty point reference with intensity Q located at the point $(0, 0, h_s)$

$$C(x, y, z)=\frac{Q\delta(y)\delta(z-h_s)}{U(x)}, \quad x=0,\ 0\le h_s\le H \qquad (2)$$

Where δ is the Dirac delta function, h_s is the stack height and H is the mixing height.

(2) Far off from the point source in cross wind direction the concentration of pollutants is zero. i.e.,

$$C(x, y, z)=0 \quad \text{when } y\to\pm\infty \qquad (3)$$

(3) The pollutants are reflected at the ground surface. i.e.,

$$\frac{\partial C}{\partial Z}=0 \quad \text{at } z=0 \qquad (4)$$

(4) There is some diffusion flux at the upright distance H from the ground surface.

$$k_z\frac{\partial C}{\partial z}=V_dC \quad \text{at } z=H \qquad (5)$$

In the present work, the large scale wind velocity is taken as constant. i.e., $u=U_0$. It is presume that the parallel mesoscae wind fluctuate in the similar vertical manner as u. The vertical mesoscale air current $w(z)$ can be identified by integrating the continuity equation

$$u_e=-aU_0x$$

$$w_e=aU_0z$$

a is the proportionality constant.

$$U(x)=u+u_e=U_0(1-ax)$$

$$w_e=aU_0z$$

3. METHOD OF SOLUTION

Prior to solution of Eq. (1) narrating the distribution of impurities and the boundary conditions are made non dimensional by using the following dimension less parameters:

$$x^*=\frac{K_{z0}x}{U_0H^2},\quad y^*=\frac{y}{H},\quad z^*=\frac{z}{H},\quad U^*=\frac{U}{U_0},$$

$$C^*=\frac{U_0H^2C}{Q},\quad \beta^*=\frac{K_y}{K_{z0}},\quad \gamma^*=\frac{K_z}{K_{z0}},$$

$$\delta(y^*)=H\delta(y),\quad a^*=\frac{U_0H^2a}{K_{z0}},\quad N^*=\frac{HV_d}{K_z}$$

Where U_0 is the allusion air current velocity and K_{z0} is the allusion diffusivity. On removing asterisk $(*)$, the Eq. (1) and the boundary conditions (2)–(5) are put in the non dimensional form as given below.

$$(1-ax)\frac{\partial C}{\partial x}+az\frac{\partial C}{\partial z}=\beta\frac{\partial^2 C}{\partial y^2}+\gamma\frac{\partial^2 C}{\partial z^2}-\alpha C \qquad (6)$$

$$C=\frac{\delta(y)\delta(z-h_s)}{(1-ax)} \quad \text{at } x=0 \qquad (7)$$

$$C=0 \quad \text{when } y\to\pm\infty \qquad (8)$$

$$\frac{\partial C}{\partial z}=0 \quad \text{at } z=0 \qquad (9)$$

$$\frac{\partial C}{\partial z}=NC \quad \text{at } z=1 \qquad (10)$$

We solve Eq. (6) with the boundary conditions (7)–(10) by using Fourier transform method. Taking Fourier transform of Eq. (6) with respect to y, we get

$$(1-ax)\frac{\partial \bar{C}}{\partial x}+p^2\beta\bar{C}=\gamma\frac{\partial^2\bar{C}}{\partial z^2}-az\frac{\partial\bar{C}}{\partial z}-\alpha\bar{C} \qquad (11)$$

Where $\bar{C}=\bar{C}(x, p, z)$ is the Fourier transform of $C=C(x, y, z)$ with respect to y and p is the corresponding

Fourier transform parameter. Taking Fourier transform of (7)–(10), the boundary conditions become:

$$\bar{C} = \frac{\delta(z-h_s)}{(1-ax)}, \quad x = 0 \tag{12}$$

$$\bar{C} = 0, \quad y \to \pm\infty \tag{13}$$

$$\frac{\partial \bar{C}}{\partial z} = 0, \quad z = 0 \tag{14}$$

$$\frac{\partial \bar{C}}{\partial z} = N\bar{C}, \quad z = 1 \tag{15}$$

Equation (11) is solved by variable separable method. Assume the trial solution as

$$\bar{C} = X(x)Z(z) \tag{16}$$

Where $X(x)$ is a function of only x and $Z(z)$ is a function of only z.

Using Eq. (16) in Eq. (11) we get the following two ordinary differential equations:

$$\frac{(1-ax)}{X}\frac{dX}{dx} + (p^2\beta + \alpha + \lambda^2) = 0 \tag{17}$$

$$\gamma\frac{d^2Z}{dz^2} - az\frac{dZ}{dz} + \lambda^2 Z = 0 \tag{18}$$

Where λ^2 is a separation constant.

Solution of Eqs. (17) and (18) are respectively given by

$$X = C_1(1-ax)^{p^2\beta+\alpha+\lambda^2/a} \tag{19}$$

$$Z = a_0 f(z) + a_1 g(z) \tag{20}$$

Where a_0, a_1 and C_1 are arbitrary constants and

$$f(z) = 1 - \frac{\lambda^2}{2!\gamma}z^2 - \frac{\lambda^2(2a-\lambda^2)}{4!\gamma^2}z^4 - \frac{\lambda^2(2a-\lambda^2)(3a-\lambda^2)}{6!\gamma^3}z^6 - \cdots \tag{21}$$

$$g(z) = z + \frac{(a-\lambda^2)}{3!\gamma}z^3 + \frac{(a-\lambda^2)(3a-\lambda^2)}{5!\gamma^2}z^5 + \frac{(a-\lambda^2)(3a-\lambda^2)(5a-\lambda^2)}{7!\gamma^3}z^7 + \cdots \tag{22}$$

Therefore, substituting the values of $X(x)$ from Eq. (19) and $Z(z)$ from Eq. (20) in Eq. (16), we get

$$\bar{C} = (1-ax)^{p^2\beta+\alpha+\lambda^2/a}[a_0 f(z) + a_1 g(z)] \tag{23}$$

Without any loss of generality, C_1 is taken as 1. Applying the boundary conditions $\partial\bar{C}/\partial z = 0$, $z = 0$ and $\partial\bar{C}/\partial z = N\bar{C}$, $z = 1$ we get

$$N = \frac{f'(1)}{f(1)} \tag{24}$$

Again, using the boundary condition $\bar{C} = \delta(z-h_s)/(1-ax)$, $x = 0$ and as well as applying

$$\int_0^1 \delta(z-h_s)f_n(z)\,dz = f_n(h_s) \quad \text{and}$$

$$\int_0^1 z^p f_m(z)f_n(z)\,dz = 0, \quad m \neq n$$

the solution is given by

$$\bar{C} = (1-ax)^{p^2\beta+\alpha+\lambda^2/a}\frac{f(h_s)}{p}f(z) \tag{25}$$

Where $p = \int_0^1 f^2(z)dz$.

Finally, taking the inverse Fourier transform of (23), we get

$$C = 0.28209\sqrt{\frac{a}{\beta\log(1/1-ax)}}(1-ax)^{\alpha+\lambda^2/a} \times \frac{f(h_s)f(z)}{p}\exp\left(\frac{ay^2}{4\beta\log(1-ax)}\right) \tag{26}$$

4. RESULTS AND DISCUSSION

In order to study the dispensation of air impurities, congregation profiles have been calculated for various conditions using non dimensional equation (24). The parametric values used in the analysis are as follows.

$$\beta = 10, \quad \gamma = 1, \quad vd = 0.001, \quad hs = 0.2,$$
$$kz = 0.01, \quad H = 1$$

In Figure 1 the congregation profile against the downwind distance $(0 \leq x \leq 5)$ is plotted, keeping the crosswind distance fixed at the value $(y = 0)$ for various values of removal rate $(\alpha = 0, 1, 2)$. From the graph, it is observed that the congregation is high near the origin and goes on decreases as downwind distance increases and it is negligible for certain value of x. Also we noticed that the congregation of air impurities is high in the absence of removal rate compared with the presence of removal rate.

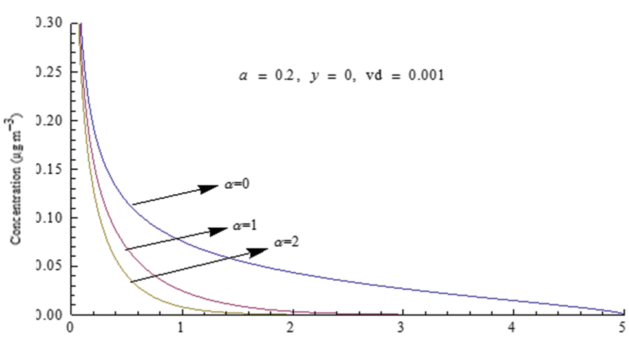


Fig. 1. Variation of congregation profile with downwind distance for different removal rates. $(\alpha = 0, \alpha = 1, \alpha = 2)$.

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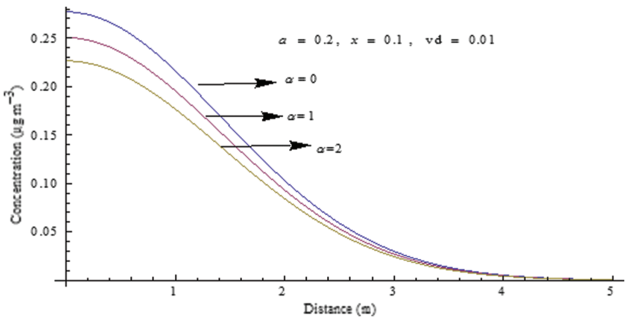

Fig. 2. Variation of concentration profile with crosswind distance for different removal rate. ($\alpha = 0$, $\alpha = 1$, $\alpha = 2$).

In Figure 2 the concentration profile is plotted against the crosswind distance ($0 \leq y \leq 5$) for different removal rates. From the graph wenoticed that the congregation of air impurities is decreased with increase of downwind distance. Also it is observed that the concentration of pollutants is more in the obsence of removal rate compared with presence of removal rate.

In Figure 3 the concentration profile is plotted against the vertical distance ($0 \leq z \leq 5$). From the graph it is observed that as the vertical distance increases the concentration decreases. Here also we observed that the concentration of the pollutants is more in the absence of removal mechanism. As the removal mechanism increases the concentration of the pollutants decreases.

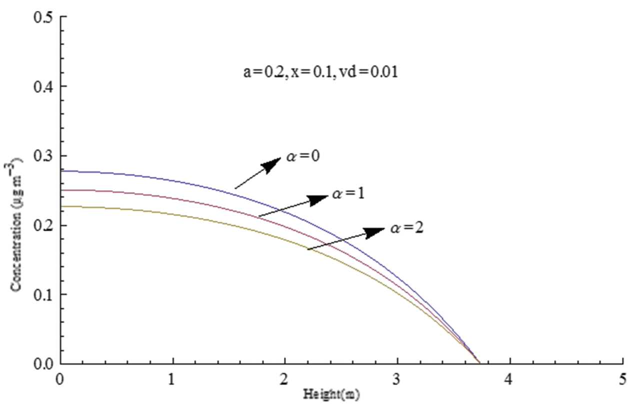

Fig. 3. Variation of concentration profile with vertical distance for different removal rates. ($\alpha = 0$, $\alpha = 1$, $\alpha = 2$).

5. CONCLUSION

The consequences of removal mechanism like first order chemical reaction on the distribution of air impurites in the presence of local air current for the raised point source is pressented. The results of this model have been analysed for the dispensation of air impurities in a civic zone for downwind, crosswind and vertical distance. The congregation of air impurities decreases with increase of these distances. Also it shows that the congregation of air impurities is less in the presence of removal mechanism when compared in the absence of removal mechanism.

References

1. J. F. Griffiths, Problems in urban air pollution, *AIAA 8th Aerospace Science Meeting*, AIAA, New York **(1970)**, pp. 70–112.
2. Vijai Shanjer Verma, *Int. J. Acad. Phy. Sci.* 15, 43 **(2011)**.
3. V. S. Verma, Uma Srivatsava, and P. S. Bhandari, *Int. J. Sci. and Research* 4, 1904 **(2015)**.
4. Maithili Sharan, M. P. Singh, and A. K. Yadav, *Atmospheric Environment* 29, 1137 **(1995)**.
5. Maithili Sharan and A. K. Yadav, *Atmospheric Environment* 30, 1209 **(1996)**.
6. T. J. Chandler, *Atmospheric Environment* 2, 619 **(1968)**.
7. J. F. Dilley and K. T. Yen, *Atmospheric Environment* 5, 843 **(1971)**.
8. M. Venkatachalappa, Sujitkumar Khan, and Khaleel Ahmed G. Kakamari, *Proc. Indian Natn. Sci. Acad.* 69, 745 **(2003)**.
9. C. Pandurangappa and K. Lakshminarayanachari, *International Journal of Research in Engineering and Technology* 1, 20 **(2012)**.
10. K. Lakshminarayanachari, K. L. Sudheer Pai, M. Siddalinga Prasad, and C. Pandurangappa, *Atmospheric Pollution Research* 4, 106 **(2013)**.
11. K. Lakshminarayanachari, C. M. Suresha, M. Siddalinga Prasad, and C. Pandurangappa, *International Journal of Environmental Sciences* 5 **(2014)**.

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Research paper

Three-Dimensional Analytical Model for the Dispersion of Air Pollutants Emitted from Elevated Point Source with Mesoscale Wind

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Abstract

A three-dimensional analytical method is evolved to analyze the distribution of air impurities released from raised point of reference in the civic zone in the existence of mesoscale wind along with huge air current. The huge air current and eddy diffusivities are taken as constants. The purpose of developing a model is for better understanding of the physical, chemical and dynamical properties of air pollution and meteorology. The analytical model is solved using the Fourier transform technique, variable separable method and series solution. In an urban area, mesoscale wind prevents the distribution of air impurities in the aerosphere. As a result of this there will be rise in aggregation of impurities in the aerosphere. The aggregation of air impurities is less for the point source which is at the origin in the existence of mesoscale wind as indicated by the research investigation.

Keywords*: *Analytical method; mesoscale wind; variable wind; point source; pollutant dispersion.

1. Introduction

The natural resources supporting life are air, water, soil and solar energy. Man can survive for a few weeks without food, for a few days without water, but without air, for only a few minutes. Hence air is considered as one of the most vital and precious natural resources. Today most of the natural resources are polluted. The increase in population, transportation and industrialization has resulted in the increase of all kinds of pollutants in the environment. The natural capacity of the environment to endure and maintain development has dwindled in the visage of the ever-increasing expulsion of pollutants. The control of air pollution is one of the difficult tasks facing an environmental manager. Both soil and water can be confined and gathered at one place while air cannot be gathered and confined in one place. Hence the control technologies for air pollution have to be at the source before the release of pollutants in to the air. To protect ourselves from contaminants released to the atmosphere, it is better to understand the physical phenomenon involved in the atmospheric pollutant dispersion. In order to reduce the pollutants emission in to the atmosphere it is necessary to monitor air quality constantly. Therefore, precise modeling of pollutants concentration near earth surroundings is significantly important.

Urban heat island produces local wind known as mesoscale wind. It has to be taken into explanation beside the large scale air current to forecast the concentration of air impurities. Griffiths [1]observed that huge air current is insufficient to forecasts atmospheric impurities in civic zone. An analytical model on distribution of air impurities by considering constant air current velocity was proposed by Verma VS[2] Varma et al [3]studied the distribution of air impurities with variable wind velocity. Sharan et al. [4], [5]outline the mathematical modeling structure of atmospheric distribution. All these models are analytical models and they are not measured the consequences of mesoscale air current in an urban zone. Chandler [6] pointed out that close to the centre of heat islet the erect amalgamation would be enhanced by mesoscale air current. Dilley and Yen [7] have presented a mathematical method for the distribution of atmospheric pollutant in the presence of mesoscale wind for a line source. Pandurangappa C and Lakshminarayanachark K [8], [9], [10], [11], [12], [13], [14], [15}, [16] have made a thorough study on the consequences of mesoscale wind on the distribution of air impurities using numerical model. Some of the authors have studied the health disorders due to air pollution. Pope et al.[17] showed that respirable particulate air pollution causes persistent respiratory disease and also increases the mortality rate. However these works did not compact with the consequences of mesoscale air current for the point source. In view of this, we develop an analytical method for the distribution of air pollutants released from an elevated point reference in the presence of mesoscale air current generated by city heat island.

2. Mathematical model formulation

Basic equation describing distribution of pollutants in the ambience based on the gradient transport theory is

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=\frac{\partial}{\partial x}\left(k_x\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial C}{\partial z}\right)-\alpha C \quad (1)$$

Where C is the congregation of pollutants in the atmosphere u, v, w and k_x, k_y, k_z are the wind speed and diffusivities in x, y, zdirections. $\alpha$is the removal rate of the air pollutant due to some natural mechanism.

The corporal problem consists of a point source elevated at a distance h_s mts from the ground. We suppose that the air impurities are emitted at steady speed from the point reference. The pollutants are transited parallel to huge scale air current and horizontally plus vertically by the local air current called mesoscale wind. The centre of warm islet is considered at origin, $(x = 0,\ y = 0)$. The concentration distribution was computed up to the preferred downwind length $l = 5\ km$ i.e., $0 \le x \le l$.

While formulating mathematical model the following assumptions are made:

(i) The pollutants are emitted from the point source at a constant rate.

(ii) Steady state conditions are measured, i.e., $\frac{\partial C}{\partial t} = 0$.

(iii) x −axis is slanting towards the mean wind $(u = U and v = 0)$.

(iv) The parallel transfer by the air current dominates over parallel distribution, i.e., $u\frac{\partial C}{\partial x} >> \frac{\partial}{\partial x}\left(k_x\frac{\partial C}{\partial x}\right)$.

(v) Removal mechanism of pollutants is neglected i.e., $\alpha = 0$.

Under the above assumptions, equation (1) takes the following form.

$$U(x)\frac{\partial C}{\partial x}+w(z)\frac{\partial C}{\partial z}= k_y\frac{\partial^2 C}{\partial y^2}+ k_z\frac{\partial^2 C}{\partial z^2} \quad (2)$$

Where x, y, z are the Cartesian co-ordinates, $U(x)$ is the air current velocity in the $x$direction which varies with the downwind distance. It is assumed in the form

$U(x) = U_0(1 - ax)$,

whereU_0 is the mean wind velocity.

W(z) = $U_0 az$ represents wind speed in the z direction. Typically $k_y > k_z$ in the atmosphere.

The boundary conditions for the equation (2) are taken as follows:

(1) The pollutant is liberated from lofty point reference with intensity Q located at the point $(0, 0, h_s)$

$$C(x,y,z) = \frac{Q\delta(y)\delta(z-h_s)}{U(x)}, \qquad x = 0,\ 0 \le h_s \le H \quad (3)$$

Where δ is the Dirac delta function, h_s is the stack height and H is the mixing height.

(2) Far off from the point source in cross wind direction the concentration of pollutants is zero. i.e.,

$$C(x, y, z) = 0 \text{ when } y \to \pm\infty \quad (4)$$

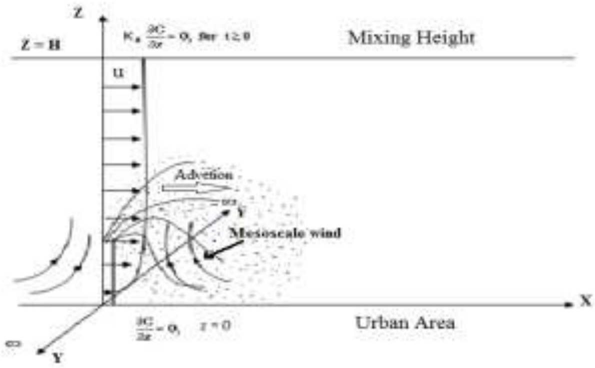
(3) The pollutants are reflected at the ground surface. i.e.,

$$\frac{\partial C}{\partial z} = 0 \quad at \quad z = 0 \quad (5)$$

(4) There is no diffusion flux at the vertical height H from the ground surface.

$$k_z\frac{\partial C}{\partial z} = 0 \text{ at } z = H \quad (6)$$

In the present work, the large scale wind velocity is taken as constant. i.e., $u = U_0$. It is presume that the parallel mesoscale wind fluctuate in the similar vertical manner as u. The vertical



mesoscale air current $w(z)$ can be identified by integrating the continuity equation

$u_e = -aU_0 x$

$w_e = aU_0 z$

$a$is the proportionality constant.

$U(x) = u + u_e = U_0(1 - ax)$

$w_e = a\,U_0\,z$

3. Method of solution

The partial differential equation (2) describing the distribution of air pollutant and the boundary conditions are made non dimensional by using the following dimension less parameters:

$$x^* = \frac{K_{z0}x}{U_0 H^2},\quad y^* = \frac{y}{H},\quad z^* = \frac{z}{H},\quad U^* = \frac{U}{U_0},\quad C^* = \frac{U_0 H^2 C}{Q},$$
$$\beta^* = \frac{K_y}{K_{z0}},\ \gamma^* = \frac{K_z}{K_{z0}},\ \delta(y^*) = H\delta(y),\quad a^* = \frac{U_0 H^2 a}{K_{z0}}. \quad (2)$$

Where U_0 is the reference wind velocity and K_{z0} is the reference diffusivity. Equation (2) and the boundary conditions (3) - (6) are put in the non dimensional form on dropping asterisk (*).

$$(1-ax)\frac{\partial C}{\partial x} + az\frac{\partial C}{\partial z} = \beta\frac{\partial^2 C}{\partial y^2} + \gamma\frac{\partial^2 C}{\partial z^2} - \alpha C \quad (7)$$

$$C = \frac{\delta(y)\delta(z-h_s)}{(1-ax)} \quad at \quad x = 0 \quad (8)$$

$$C = 0 \quad \text{when} \quad y \to \pm\infty \quad (9)$$

$$\frac{\partial C}{\partial z} = 0 \quad at \quad z = 0 \quad (10)$$

(3)

$$\frac{\partial C}{\partial z} = 0 \quad at \quad z = 1 \quad (11)$$

We solve equation (7) along with the equations (8) - (11) by using Fourier transform method. Taking Fourier transform of equation (7) with respect to y, we get

(4)

$$(1-ax)\frac{\partial \bar{C}}{\partial x} + p^2\beta\bar{C} = \gamma\frac{\partial^2 \bar{C}}{\partial z^2} - az\frac{\partial \bar{C}}{\partial z} \quad (12)$$

Where $\bar{C} = \bar{C}(x, p, z)$ is the Fourier transform of $C = C(x, y, z)$ with respect to y and p is the corresponding Fourier transform parameter. Taking Fourier transform of (8) – (11), the boundary conditions become:

$$\bar{C} = \frac{\delta(z-h_s)}{(1-ax)} \quad at \quad x = 0 \quad (13)$$

$$\bar{C} = 0 \quad when \quad y \to \pm\infty \quad (14)$$

$$\frac{\partial \bar{C}}{\partial z} = 0 \quad at \quad z = 0 \quad (15)$$

$$\frac{\partial \bar{C}}{\partial z} = 0 \qquad at \qquad z = 1 \tag{16}$$

Again, equation (12) is solved by variable separable method. Assume trial solution as,

$$\bar{C} = X(x)Z(z) \tag{17}$$

Where X(x) is a function of only x and $Z(z)$ is a function of only z.
Using equation (17) in equation (12) we obtain two ordinary differential equations:

$$\frac{(1-ax)}{X}\frac{dX}{dx} + (p^2\beta + \lambda^2) = 0 \tag{18}$$

$$\gamma\frac{d^2Z}{dz^2} - aZ\frac{dZ}{dz} + \lambda^2 Z = 0 \tag{19}$$

Where λ^2 is a separation constant.
Solution of equations (18) and (19) are respectively given by

$$X = C_1(1-ax)^{\frac{p^2\beta+\lambda^2}{a}} \tag{20}$$

$$Z = a_0 f(z) + a_1 g(z) \tag{21}$$

Where a_0, a_1 and C_1 are arbitrary constants and

$$f(z) = 1 - \frac{\lambda^2}{2!\,\gamma}z^2 - \frac{\lambda^2\,(2a-\lambda^2)}{4!\,\gamma^2}z^4 - \frac{\lambda^2\,(2a-\lambda^2)(3a-\lambda^2)}{6!\,\gamma^3}z^6 - \cdots\cdots\cdots$$

$$g(z) = z + \frac{(a-\lambda^2)}{3!\,\gamma}z^3 + \frac{(a-\lambda^2)\,(3a-\lambda^2)}{5!\,\gamma^2}z^5 + \frac{(a-\lambda^2)(3a-\lambda^2)\,(5a-\lambda^2)}{7!\,\gamma^3}z^7 + \cdots\cdots\cdots$$

Therefore, substituting the values of $X(x)$from equation (20) and $Z(z)$ from equation (21) in equation (17), we get

$$\bar{C} = (1-ax)^{\frac{p^2\beta+\lambda^2}{a}}[a_0 f(z) + a_1 g(z)] \tag{22}$$

Where C_1 is taken as 1, without any loss of generality. Now, using the boundary conditions

$$\frac{\partial \bar{C}}{\partial z} = 0, z = 0 \; and \; \frac{\partial \bar{C}}{\partial z} = 0, z = 1 \text{we get} N = \frac{f'(1)}{f(1)} \tag{23}$$

Again, using the boundary condition $\bar{C} = \frac{\delta(z-h_s)}{(1-ax)}$, $x = 0$ and as well as applying
$\int_0^1 \delta(z-h_s) f_n(z)dz = f_n(h_s)$and
$\int_0^1 z^p f_m(z) f_n(z)dz = 0, m \neq n,$ the solution is given by

$$\bar{C} = (1-ax)^{\frac{p^2\beta+\lambda^2}{a}}\frac{f(h_s)}{p}\; \mathrm{f(z)} \tag{24}$$

Where $p = \int_0^1 f^2(z)dz$.
Finally, taking the inverse Fourier transform of (24), we get

$$\mathrm{C} = 0.28209\sqrt{\frac{\mathrm{a}}{\beta\log\left(\frac{1}{1-\mathrm{a\,x}}\right)}}(1-\mathrm{ax})^{\frac{\lambda^2}{\mathrm{a}}}\frac{\mathrm{f(h_s)\;f(z)}}{\mathrm{p}}\;\exp\left(\frac{\mathrm{a\,y^2}}{4\,\beta\log(1-\mathrm{a\,x})}\right) \tag{25}$$

4. Results and discussion

A three-dimensional analytical method to calculate surrounding atmospheric congregation of impurities all along the down-wind and vertical distance discharged from an elevated point reference is presented. The problem was solved by the method of separation of variables, Fourier transform and series solution has been used. By drawing graphs we analyze the result for pollutant concentration with and without mesoscale wind for different heights ($z = 0.3, z = 0.6, z = 0.9$) in the direction of wind blowing. It is observed that the concentration decreases as x increases. It is also observed that the concentration of pollutants is more when the crosswind distance $y = 0$ and $x = 0$, when compared with the concentration at crosswind distance $y = 2$ and $x = 0$, because the source is at $y = 0$.
We find the consequences of various parameters on distribution of air impurities the congregation profile in non-dimensional form is calculated with the help of equation (25).
The parametric values used in the analysis are:
$\beta = 10, \quad \gamma = 1, h_s = 0.2, \quad k_z = 0.01$
The figures 1, 2 and 3 demonstrates that the rate of change of concentration profile in the direction of wind blowing for different crosswind distances in the presence and absence of mesoscale wind at the upright distance $z = 0.3, 0.6, and\ 0.9$. From the graphs we observe that concentration decreases as downwind distance x increases. It is also observed that the concentration of pollutants is more when the crosswind distance $y = 0$ and $x = 0$, when compared with the concentration at crosswind distance y = 2 and x = 0. This is because the point source is kept at $x = 0, y = 0$and $z = h_s$. Also we observed that the concentration of pollutants is less in the presence of mesoscale air current in contrast to the absence of mesoscale air current. The reason for this is the horizontal constituent of mesoscale air current against the large scale air current for x > 0.
Figure 4 (a) and 4(b) demonstrates the concentration profile with downwind distance for different crosswind distances at a vertical distance z = 0.3. The figure depicts that the concentration decreases for the increasing values of cross wind distance as we move to the left or right of the origin.
Figure 5 demonstrates the concentration versus crosswind distance in the presence of mesoscale wind. The point source is at x = 0, y = 0 and height hs = 0.2 m. From the graph we observed that the attentiveness of the pollutant is more near the origin and decreases in bothe the directions as crosswind distance increases and it becomes zero at certain cross wind distance.
Figure 6 demonstrates the variation of concentration profile (22) with vertical distance for different crosswind distance. From the figure we observed that the concentration decreases as vertical distance increases also we observed that concentration of pollutants is less for the increase of crosswind distance.

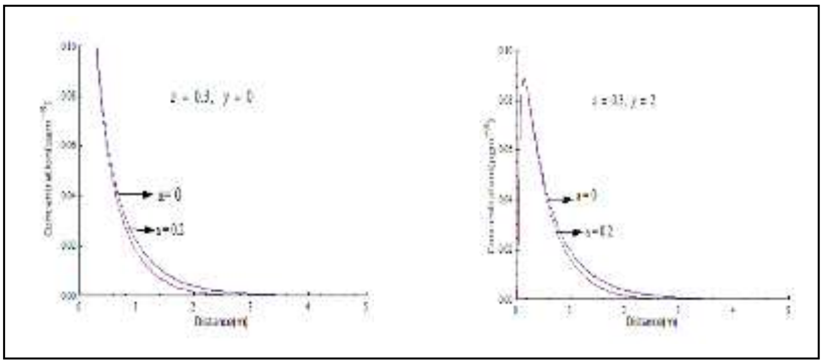


Fig 1: Variation of concentration profile with downwind distance for different crosswind distances in the presence andabsence of mesoscale wind at the vertical distance z = 0.3**.**

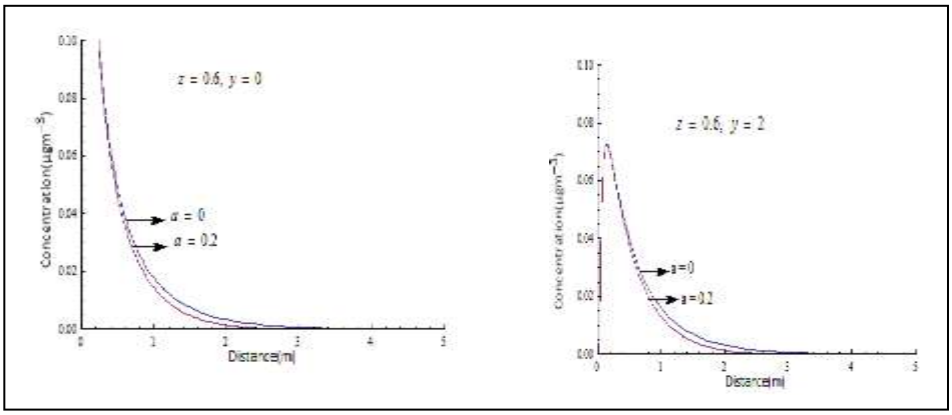


Fig. 2: Concentration versus downwind distance for different crosswind distance at the vertical distance z = 0.6

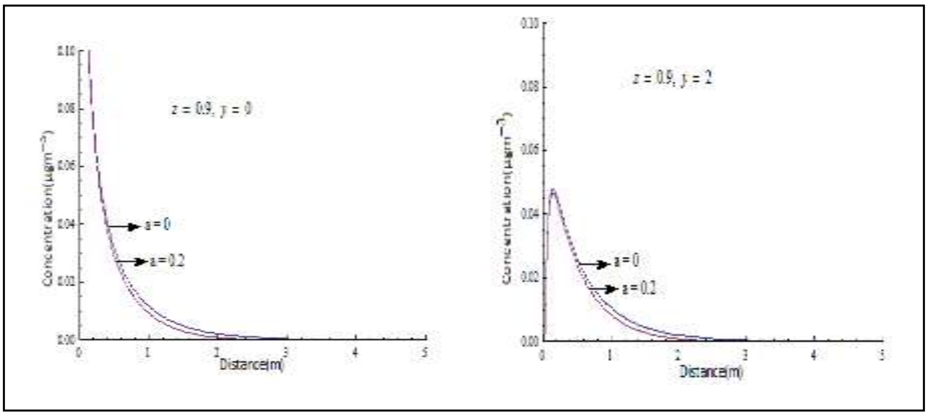


Fig. 3: Variation of concentration profile with downwind distance for difference crosswind distance at the uprightdistance z = 0.9.

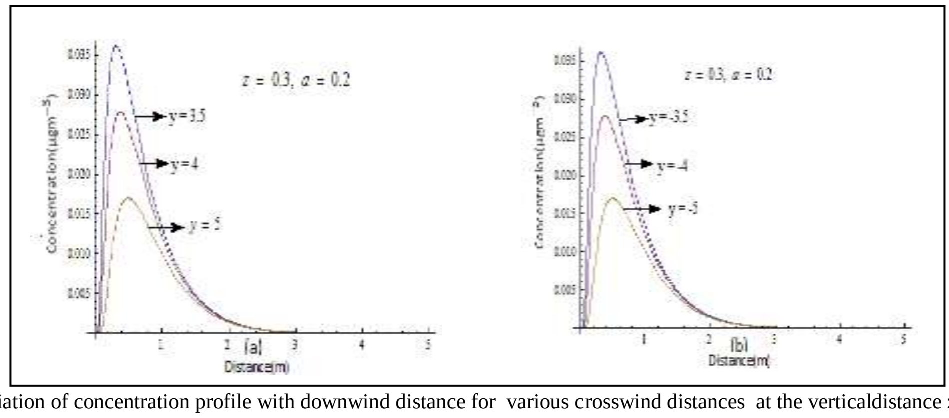


Fig. 4: Variation of concentration profile with downwind distance for various crosswind distances at the verticaldistance z = 0.3.

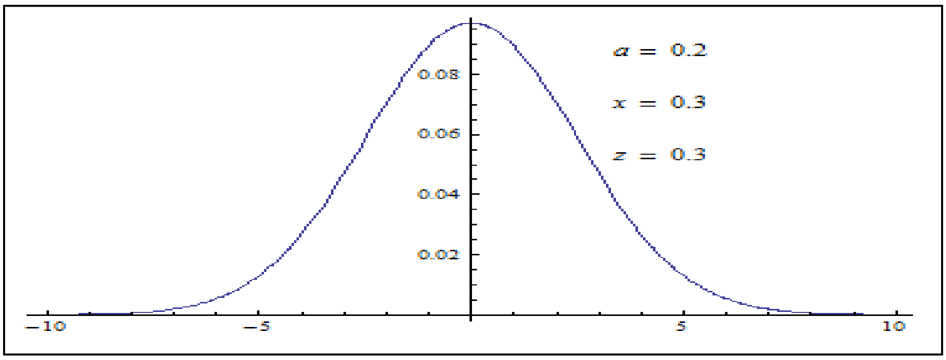


Fig. 5: Disparity of concentration profile with crosswind distance in the existence of mesoscale wind at the downwind distance x = 0.3 and vertical distance z = 0.3

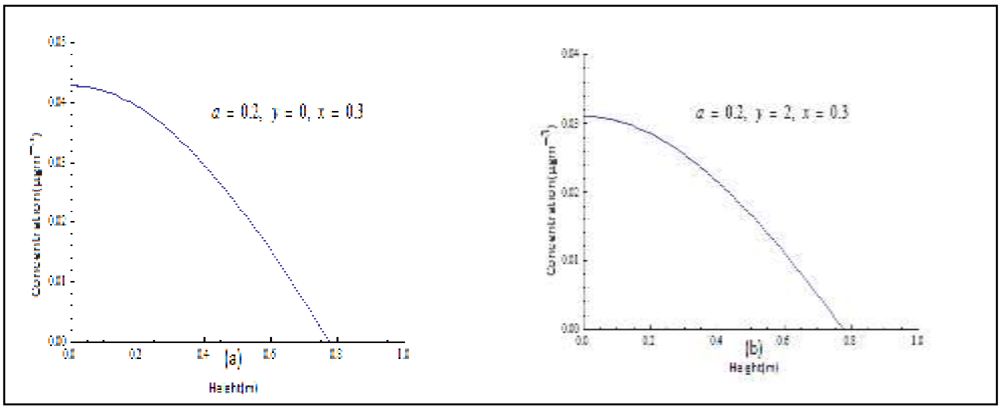


Fig. 6: Variation of concentration profile with vertical distance for different crosswind distance y = 0, y = 1, at x = 0.3.

5. Conclusion

In this model we studied the consequences of mesoscale air current on the distribution of atmospheric pollutants liberated from the point source in an urban area. We analyze the result for the distribution of air pollutants along downwind, crosswind and vertical distance. The concentration of pollutants decreases with increase in downwind distance and it becomes negligible at x =3. Also the model shows that the congregation of atmospheric impurities is less in the presence of mesoscale air current when compared in its absence. The reason for this is, the centre of heat island is at (0, 0, hs).

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References

[1]. Griffiths, J.F.," Problems in urban air pollution", in: AIAA 8th Aerospace Science Meeting, AIAA, New York, 1970, 70-112,

[2]. Verma, V.S., "An analytical approach to the problem of dispersion of an air pollutant with constant wind velocity and constant removal rate", Journal of international academy of physical sciences, 15, 2011, 43-50.

[3]. Verma, V.S., Uma Srivastava,Bhandari, P.S., "A Mathematical Model on Dispersion of Air Pollutants", International Journal of Sciences and Research, 4(12), 2015, 1904-1907.

[4]. Maithili Sharan, M.P Singh, and A.K.Yadav, "Mathematical model for atmospheric dispersion in low winds with eddy diffusivities as linear functions of downwind distance", Atmospheric Environment, 29:16, 1995, 1137-1145.

[5]. Maithili Sharan and A. K. Yadav," Atmospheric model for the dispersion of air pollutant in low wind conditions", Atmospheric Environment, 30, 1996, 1209-1220.

[6]. Chandler, T.J., "Discussion of the paper by MARSH and FPSTER, The bearing of the urban temperature field upon urban pollution patterns", Atmospheric Environment, 2, 1968, 619-62.

[7]. Dilley, J.F., Yen, K.T., "Effect of mesoscale type wind on the pollutant distribution from a line source",Atmospheric Environment, 5, 1971, 843-851.

[8]. Pandurangappa, C., Lakshminarayanachari,K., "Mathematical model for an area source and the point source in an urban area", International Journal of Research in Engineering and Technology, 1, 2012, 20-28.

[9]. Pandurangappa, C., Lakshminarayanachari,K., M. Venkatachalappa., "Effectof Mesoscale Wind on the Pollutant Emitted from an Area Source of Primary and Secondary Pollutants with Gravitational Settling velocity", International Journal of Emerging Technology and Advanced Engineering, 2, 9, 2012, 325-334.

[10]. Pandurangappa, C., Lakshminarayanachari, K., "Removal of Pollutants by Gravitational Settling and Leakage velocity with Large and Mesoscale Winds on the Pollutant Distribution Emitted from an Area Source", International Journal of Advances in Science and Technology, 4, 5, 2012, 64-95.

[11]. Pandurangappa, C., "Effect of Gravitational Settling on the Pollutant Emitted from an Area Source along with the Point Source on the Boundary", International Journal of Emerging trends in Engineering and Development, 6, 2012, 539-556.

[12]. Pandurangappa, C., "Effect of Mesoscale Wind on the Pollutant Distribution Emitted from an Area Source of Primary and Secondary Pollutants with Point Source on the Boundary", International Journal of Emerging trends in Engineering and Development, 6, 2, 2012, 539-556.

[13]. Pandurangappa, C., Krishna, S., Lakshminarayanachari, K., Sujit Kumar Khan, "Effect of Mesoscale Type Wind and Chemical Reaction On the Pollutant Distribution Emitted from a Line Source", International Journal of Emerging trends in Engineering and Development, 6, 2, 2012, 557-563.

[14]. Anjaneyulu , Y., Air Pollution And Control Technologies. Allied Publishers Pvt. Ltd. 2002.

[15]. Pasquill, F., Smith, F.B., Atmospheric Diffusion. John Wiley & Sons. 1983.